N76-19445

A THEORETICAL AND EXPERIMENTAL INVESTIGATION OF THE DYNAMIC RESPONSE OF ROLAMITE

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ABSTRACT

In the following paper, Lagrange's equation is used to derive an equation of motion for the rolamite mechanism in the ideal tight configuration; the effects of band inertia, force generation by bands of varying bending stiffness, and nonconservative forces are considered. An experimental system for investigating the dynamic response of rolamite is described. The effects of friction on the rolamite mechanism are discussed and experimentally determined values of coefficients of friction are presented.

INTRODUCTION

Rolamite is a new device having many potential applications in mechanical and electro-mechanical design. Some of these applications have already been realized and many others are currently undergoing development.[1,2] The basic rolamite geometry consists of two cylindrical rollers whose motion is restricted by a thin flexible band and two parallel guide surfaces, as shown in Figures 1 and 2. Under low tension, the system assumes the loose configuration shown in exaggerated form in Figure 1. As the tension is increased, the system finally assumes the tight geometry depicted in Figure 2. In this case, the rollers, band, and guide surfaces are in contact at the points A, B, and C; and the relative motion between the rollers or between a guide surface and a roller is pure rolling. This ideal kinematic arrangement yields a rolling suspension system with no sliding friction, and thus one of the claims made for rolamite is its low friction.

In the tight geometry (Figure 2), normal forces N are needed at the guide surface to establish equilibrium, so the roller cluster is able to support a load perpendicular to these surfaces. The magnitude of this load is proportional to the band tension and is limited by the allowable working stresses in the components. For a uniform band, the potential energy of the assembly is independent of position. However, if the bending stiffness of the band varies along its length, then the potential energy stored in the band will change with the position of the rollers. This change in potential energy induces a force on the rollers toward the position of minimum potential energy.

In the present paper, an equation of motion for the basic rolamite mechanism is derived, an experimental method for studying its dynamic response is described, and experimental results relating band tension to rolling friction are reported.

THEORY

A simple tight rolamite mechanism, as shown in Figure 2, is considered here. The tension in the band is assumed to be independent of time and position, and the rollers are assumed to suffer no deformation. Since the actual system operates at low velocities, wave propagation effects are neglected.

Since the system has dissipative forces, one must use Lagrange's equation in the form

$$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{q}_i}\right) - \frac{\partial K}{\partial q_i} = Q_i \quad , \tag{1}$$

where K is the kinetic energy of the system, q_i and Q_i are the i^{th} generalized coordinate and force, respectively. The generalized forces may be decomposed into conservative and non-conservative parts,

$$Q_i = - \frac{\partial V}{\partial q_i} + F_i \quad , \tag{2}$$

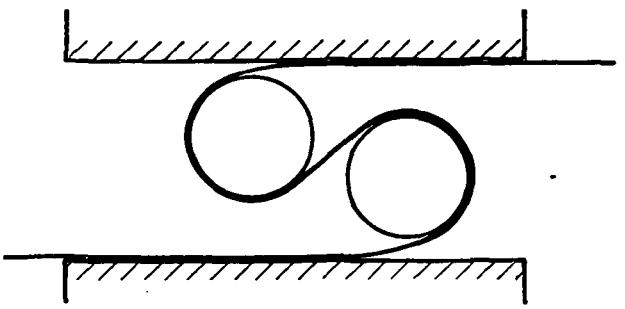

Figure 1. Rolamite Showing Exaggerated Loose Geometry

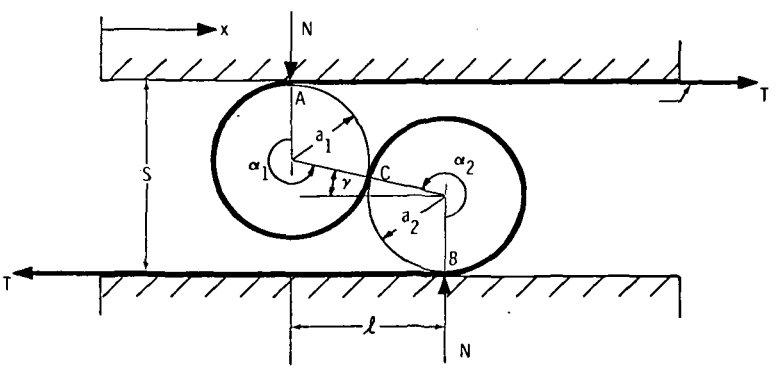

Figure 2. Rolamite in Ideal Tight Geometry

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where V is the potential energy (conservative) and F_i represents the i^{th} nonconservative force. Assuming that the translatory motion is parallel to the x-axis and that no slipping occurs between the rollers and the bands, only the generalized coordinate $q_i = x$ needs to be considered.

Kinetic Energy of the Rollers

The kinetic energy of the i^{th} roller (i = 1, 2) is given by the equation

$$K_i = \frac{1}{2} m_i v^2 + \frac{1}{2} I_i \Omega_i^2 \quad , \tag{3}$$

in which m_i is the mass of the i^{th} roller, $v = \dot{x}$ is the velocity of its center of mass, I_i its moment of inertia, and Ω_i its angular velocity. Since it is assumed that no slippage occurs, $v = \Omega_i a_i$ and, hence,

$$K_i = \frac{v^2}{2a_i^2} \left(m_i a_i^2 + I_i \right) \quad . \tag{4}$$

Kinetic Energy of the Elastic Band

At all times the point A shown in Figures 2 and 3 is the instantaneous center of rotation for the part of the band around the upper roller. Therefore, the velocity of the point P shown in Figure 3 is given by

$$\underset{\sim}{u}(\varphi) = \Omega_1 \underset{\sim}{k} \times \underset{\sim}{r} \quad , \quad \Omega_1 a_1 = v \quad ,$$

where $\underset{\sim}{k}$ is the unit vector in the z-direction. From the figure,

$$\underset{\sim}{u}(\varphi) = - \underset{\sim}{j} v \sin\varphi + \underset{\sim}{i} v(1 - \cos \varphi) \quad , \tag{5}$$

$$\underset{\sim}{u} \cdot \underset{\sim}{u} = 2v^2(1 - \cos \varphi) \quad ,$$

where $\underset{\sim}{i}$ and $\underset{\sim}{j}$ are the unit vectors in the x and y direction, respectively. The kinetic energy of this portion of the band is given by

$$K_{b1} = \int \frac{1}{2}(\underset{\sim}{u} \cdot \underset{\sim}{u}) a_1 \rho_b A d\varphi \quad ,$$

where A is the cross-sectional area of the band and ρ_b is the density of the material of which it is made. Hence,

$$K_{b1} = A \rho_b a_1 v^2(\alpha_1 - \sin \alpha_1) \quad . \tag{6}$$

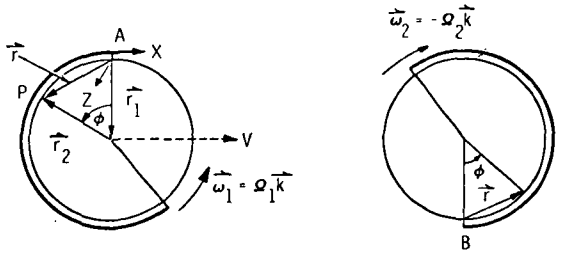

Figure 3. Auxiliary Diagrams for Kinetic Energy Determination

Similarly, for the band around the other roller, one finds that

$$K_{b2} = A\,\rho_b\,a_2\,v^2(\alpha_2 - \sin\alpha_2) \quad , \tag{7}$$

(See Figure 3b).

By adding Equations 4, 6, and 8, one finds that the total kinetic energy of the system is given by $K = I\,v^2/2$ where

$$I = \sum_{i=1}^{2} \left\{ \frac{1}{a_i^2}\left(m_i\,a_i^2 + I_i\right) + 2A\,\rho_b\,a_i\,(\alpha_i - \sin\alpha_i) \right\} \quad . \tag{8}$$

Elastic Energy

In the case of a uniform band, the bending energy and the energy of tension of the band remain constant (say $V_b + V_t = B$) throughout the motion and, therefore, do not contribute to the generalized force. For many applications, bands of variable bending stiffness are required. This variation is usually obtained by punching holes of appropriate shape in the band, as shown in Reference 2. When the band has one of these cutouts, then the bending energy of the band is decreased from its normal value by the energy required to flex a band similar in shape to the cutout. Using beam theory, the energy associated with an element of the cutout of length dx is

$$dV_c = \frac{E}{2} \cdot \frac{b^3 w}{12} \cdot \frac{(d\varphi)^2}{dx} = \frac{E}{2} \cdot \frac{b^3 w}{12} \cdot \frac{dx}{a^2} \quad , \quad w = w(x) \quad , \tag{9}$$

where b is the thickness of the band, w(x) is the width of the cutout, and a is the radius of the cylinder. In Figure 2, the origin of the x-axis is selected to correspond with the start of the cutout when the band is flat on the surface y = h. Thus, in the configuration shown,

$$V_c = \frac{Eb^3}{24\,a_1^2} \int w(x)\,dx \quad ,$$

and

$$V = B - V_c \quad , \tag{10}$$

where B is the energy that would be stored in the band if there were no cutout.

Equation of Motion for the System

When the aforegoing results are substituted into Equation 1, one obtains

$$\frac{d^2x}{dt^2} \cdot \sum_{i=1}^{2} \left\{ \frac{1}{a_i^2} \left(m_i a_i^2 + I_i \right) + 2 \rho_b a_i (\alpha_i - \sin \alpha_i) \right\} = \frac{Eb^3}{24 a_1^2} w(x) + F_x \quad , \tag{11}$$

where F_x represents the nonconservative force. From the first term on the right side, we see that the device produces an acceleration proportional to the width of the cutout.

For very thin bands there is a range of tension wherein the tight geometrical configuration exists; but the normal force at the band-roller interface is insufficient to prevent slippage, and thus the device is loose in respect to some of its kinematical properties. Depending on the coefficient of friction for the band-roller interface, there is a value of tension below which slippage occurs. At this value the relation $V = \Omega_i a_i$ given after Equation 3 becomes inapplicable and must be replaced by $kV = \Omega_i a_i$, where $0 \leq k \leq 1$. As a consequence of this change, I_i should be replaced by $k^2 I_i$ in Equations 4, 8, and 11. The effect of slippage is thus seen to be a reduction in the effective inertia of the device.

EXPERIMENT TO MEASURE FRICTIONAL FORCE

To determine the nonconservative force, it is necessary to appeal to experiments. One may assume that the force F_x depends on (1) hysteresis losses in the flexing of the band, (2) viscous friction, and (3) friction losses at contacting surfaces, including elastic-plastic deformation of band rollers, guide surfaces, and adhesion of contacting materials.

Experimental determinations of the magnitude of band hysteresis losses have shown that in a well-designed device these are negligible when compared to the overall friction of the rolamite mechanism.[3] That the viscous friction is also very slight will be demonstrated later in this paper. Thus, it was concluded that the dominant nonconservative force in rolamite devices of the sort under consideration depends on friction at the contact surfaces and corresponds to a constant value of F_x in Equation 11.

By operating the rolamite under well-defined conditions, it is possible to relate the variation of actual dynamic response from that predicted by a friction-free theory to these friction forces. An experiment was proposed wherein the rollers were allowed to fall vertically under the influence of gravity for a fixed distance. The only variable was the band tension, which influences the normal force and contact pressures in the rolamite.

For the above conditions, the equation of motion takes the form

$$I \ddot{x} = W - F_x \quad , \tag{12}$$

where I is the total inertia of the moving mass, W is the total weight of the moving mass (rollers and band), and F_x is the friction force resisting motion. The moving mass should include not only the rollers and the band on the rollers but also a small portion of band of length ℓ shown in Figure 2. This portion of band will undergo effective translation with a corresponding decrease in potential energy due to the downward motion of the roller cluster with the rolamite in the vertical orientation. If the friction force is constant in a given test, the acceleration will also be constant, but will be less than the frictionless ideal case. By measuring the time required for the rollers to fall a fixed distance, the acceleration can be determined. The weight and inertia of the moving mass are known and the friction force can be calculated from Equation 12. The coefficient of friction f was chosen to be the ratio of the frictional force to the normal force between the rollers and the guide surfaces.

Experimental Technique

The displacement history of the rollers was obtained by optical means, thus avoiding the introduction of external forces that could influence the motion of the system. An overall schematic of the experimental setup appears in Figure 4. A light source provides a uniform and

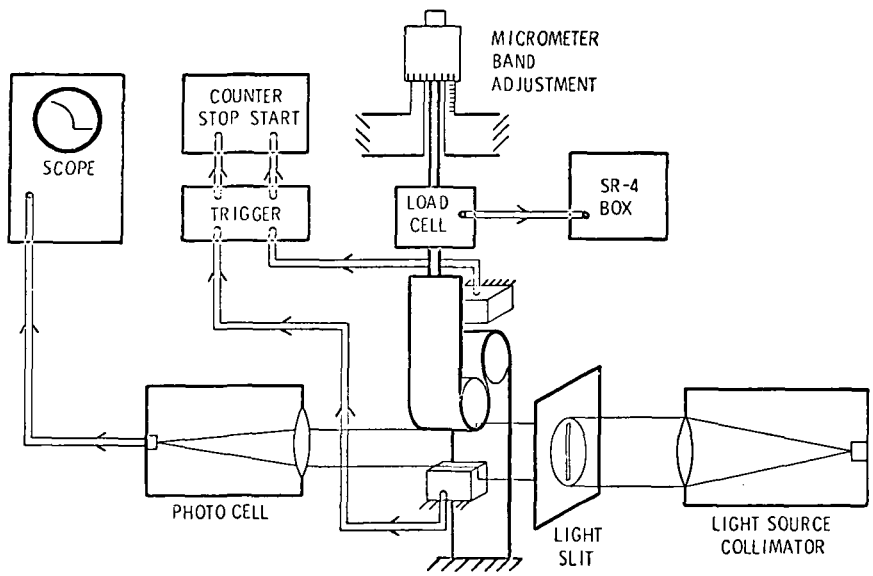

Figure 4. Schematic Representation of Experimental Arrangement

parallel beam that passes through the rolamite and is focused onto a photocell. As the rollers travel along the axis of motion, the light beam is blocked and its intensity, as measured by the photocell, is proportional to the displacement of the rollers. This system was calibrated by closing the light slit with a shutter attached to a micrometer. The calibration data, as well as the rolamite displacement data, is recorded on an oscilloscope and reduced by means of an optical comparator. A computer program was written to correct the rolamite displacement data for nonlinearities in the optical transducer system.

The rollers are initially held against an upper stop by means of a bracket that catches an extension on each end of the upper roller. To release the rollers the bracket is withdrawn by means of an electrical solenoid. Both the rollers and the upper stop are sufficiently rigid that no significant elastic energy is released upon separation. Trials have shown that the response of the rolamite is independent of the initial force between the roller and the upper stop.

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The action time required for the rollers to fall a fixed distance is measured independently of the displacement history. An electrical circuit using a low voltage capacitor trigger provides a microsecond long pulse to start a time interval counter when the contact between the upper roller and the upper stop is broken. When the lower roller strikes a lower stop, the trigger circuit provides an electrical pulse that stops the counter. The lower stop is adjustable so that the distance that the rollers are allowed to fall can be accurately controlled.

The lower end of the band is rigidly fixed to the rolamite frame while the other end of the band passes through a slit in the frame and is attached through a load cell to a micrometer adjusting stress to enable precise adjustment of the band tension. The load cell incorporates a strain gage instrumented cantilever beam as the sensing element and has an upper load limit of 10 pounds. When initially installed or adjusted, the band tension is nonuniform due to the band being loaded by static friction at the contact points. Before collecting data in any given configuration, care was taken to ensure a uniform band tension. This was achieved only by moving the cylinders up and down several times. To demonstrate that the tension becomes uniform, the rigid lower attachment was replaced by a hanging weight. When the weight was first attached to the band, the load cell did not register any tension in the upper part of the band. After several cycles of operation, the tension in the upper part of the band, as measured by the load cell, was within 2 percent of that in the lower part of the band.

Configuration

The rolamite configuration chosen for the initial experimental studies employed hardened steel rollers and guide surfaces. The rollers were 0.500 inch in diameter by 0.750 inch long. For holding and positioning purposes, small extensions 0.125 inch in diameter by 0.125 inch long were made on each end of the rollers. The guide surfaces were 1.00 inch wide. Beryllium-copper foil was chosen as the band material and four thicknesses ranging from 0.0005 to 0.0020 inch were used. The foil, which has a Young's modulus of 17×10^6 psi, was heat treated to a hardness of RC 37-41, which corresponds to a yield stress of 140,000 to 170,000 psi. Bands of uniform cross section were used during this study; thus, $w(x) = 0$ in Equation 11. The distance S between the guide surfaces was fixed at 0.625 inch plus the thickness of the band being used.

This configuration was chosen to be representative, both in size and materials used, of those which seem most likely to find application in precision instruments and similar designs. The beryllium-copper band is known to have low hysteresis losses. The maximum calculated stress in the band due to combined tension, bending, and contact stresses at maximum band tension occurred in the 0.002 inch band and was less than 50 percent of the yield stress for berylliumcopper.

To confirm that the rollers did experience approximately constant acceleration, the displacement history was measured using the optical diagnostics. Figure 5 shows an example of the experimental displacement history and a comparison to theoretical results for $F_x = 0$. The results shown are for a 0.0005 inch thick band and a band tension of 0.30 pound. At this tension, no slipping occurs and the friction force F_x is small since the normal forces are low. The low magnitude of friction force is demonstrated by the close correspondence of the two results. The good agreement between the two results also demonstrates that the rolamite system has very little viscous friction, which would be independent of band tension and most evident in cases where other forms of friction are low.

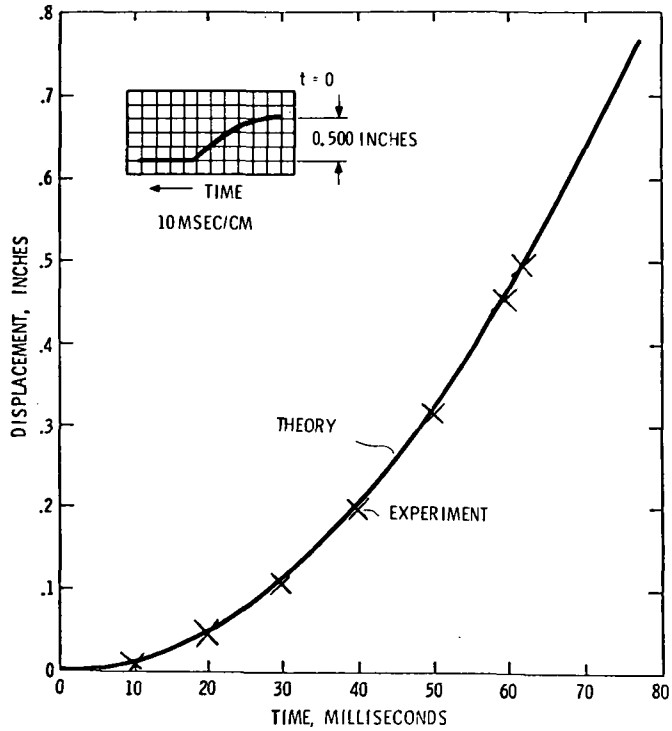

Figure 5. Displacement versus Time for Rolamite with 0.0005 Inch Thick Band

RESULTS

Figure 6 is a graph of the action time required for the roller-cluster with a 0.002 inch thick band to fall 0.500 inch for various values of tension in the band. This graph represents the average of four runs in which the action time was measured at intervals as the band tension was increased or decreased. The error bars show the maximum and minimum values of action time for the four runs at each band tension. It was found that the action time as the band tension was decreased was usually slightly less than the corresponding action time recorded previously when the band tension was being increased. This effect is due to the band "running-in" and friction being reduced. It has been shown that in the tight geometry the band and roller surfaces remate and successive remating will reduce the rolling friction. Thus, the data presented here is for rolamite which has been "run-in" to a fair degree not only by the test procedure but also as it is operated to achieve uniform band tension. Figure 7 is a plot of the average action times for the four band thicknesses used. The data scatter for all of the runs was similar and the error bars were omitted for clarity.

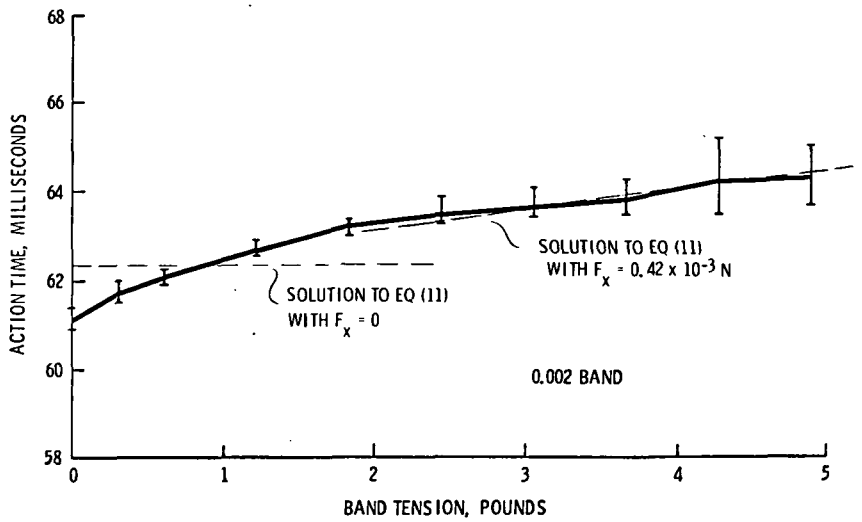

Figure 6. Action Time for 0.5 Inch Roller-Cluster Displacement with 0.002 Inch Band versus Band Tension

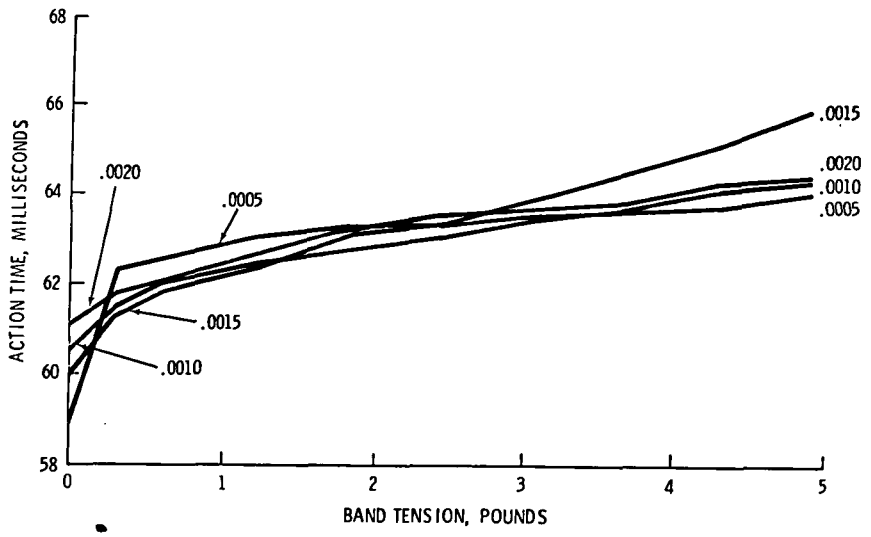

Figure 7. Action Time for 0.5 Inch Roller-Cluster Displacement versus Band Tension for Various Band Thicknesses

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The theoretical action time in the tight-frictionless condition varies slightly with band thickness from 62.18 milliseconds for the 0.0005 inch thick band to 62.37 milliseconds for the 0.002 inch band. At the low band tensions, the experimental action times were less than the theoretical values. The most probable cause of this is slippage between the band and the rollers. At the low tensions, the friction force between the two is low and relative motion is possible. This motion reduces the effective inertia of the rollers and increases the measured acceleration. The thinner bands are pulled to closer contact with the rollers at lower band tension and the effect is reduced. This reasoning is borne out by the relative action time for the various bands at low tensions. The fact that there was slipping between the band and roller at low band tension was demonstrated qualitatively by marking the roller and noting the change in position of the mark after successive operations. Above a band tension of 2.0 pounds for the 0.002 band, there was no slipping. For the 0.0005 band, there was no slipping for band tensions above 0.3 pound.

Calculations based on a geometry and force analysis for the loose rolamite configuration[4] show that at a band tension of 2.5 pounds for the 0.002 inch band there is 0.0003 inch displacement of roller centerline because of loose geometry. However, for the thinner bands, much smaller tensions would be required; for the 0.0005 inch band, only 0.04 pound tension would provide the same tight geometry. But at this very low band tension, the contact pressure between the band and roller would be very small and there would not be sufficient friction between the two to prevent sliding. Thus, the operation of the rolamite depends on the configuration being not only geometrically tight but also kinematically tight as well. Criteria to be satisfied in order that a given rolamite configuration be both geometrically and kinematically tight have not been established. Such criteria are needed for a complete description of the dynamic response.

Figure 8 is a plot of the friction coefficients calculated for the action times shown in Figure 6. It can be seen that, above a critical band tension, the coefficient of friction is

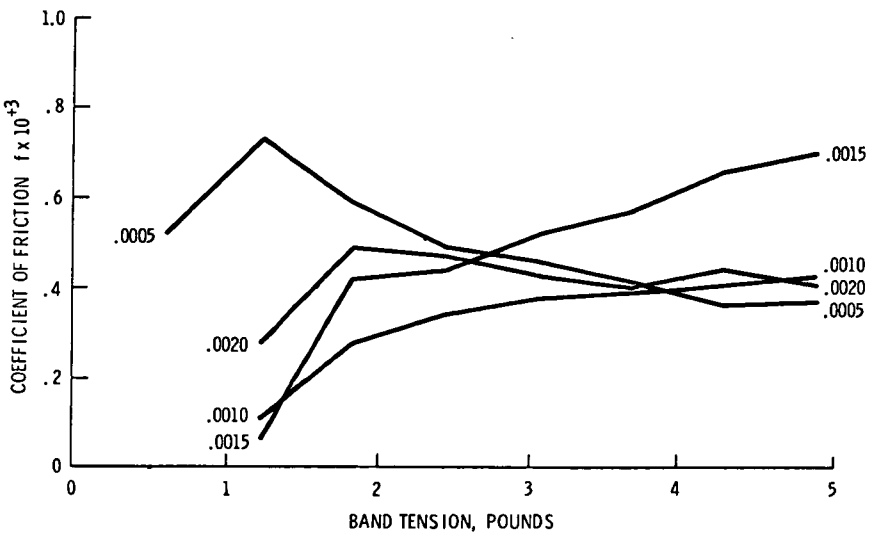

Figure 8. Coefficient of Friction versus Band Tension for Various Band Thicknesses

reasonably constant. This critical band tension is lower for the thin band than it is for the thick bands. The coefficient of friction appears to be reasonably independent of band thickness. The coefficients of friction obtained here are significantly lower than the published values for rolling friction and standard rolling type bearings, particularly at low loads.

It can be concluded that, for values of band tension at which slipping between the band and rollers does not occur and the rolamite configuration may be considered to be tight, the coefficient of friction is relatively independent of both band thickness and band tension. The values for dynamic friction obtained here confirm the results given by Wilkes[5] for quasistatic motion. The procedures developed provide a simple means of measuring the dynamic response of the rolamite mechanism and comparing such response to theoretical predictions.

Results of further investigations to determine the validity of the simple theory, particularly in respect to the effect of band cutouts and the resulting force generation, will be forthcoming.

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